Math 4575 : HW #12

Due Wednesday, April 25 by 4pm

1. Let T be a tree on n vertices. Let $P_T(k)$ denote the chromatic polynomial of T. Show that
$$P_T(k) = k(k-1)^{n-1}.$$

2. Let $G = C_n$ be a cycle of length n. Let $P_G(k)$ denote the chromatic polynomial of G. Show that
$$P_G(k) = (k-1)^n + (-1)^n(k-1).$$

3. Show that $P(k) = k^4 - 4k^3 + 3k^2$ is not the chromatic polynomial of any graph.

4. Show that if a graph G has e edges then
$$\chi(G) \leq \sqrt{2e + \frac{1}{4}} + \frac{1}{2}.$$

For the next two problems, define the Cartesian product $G\Box H$ of two graphs G and H by:
$$V(G\Box H) = \{(g,h) \mid g \in G, h \in H\}.$$
Use the notation $x \sim y$ to denote that vertex x is adjacent to vertex y. With this notation, the edges in $G\Box H$ are defined as follows. We have $(g,h) \sim (g',h')$ whenever either

(a) $g = g'$ and $h \sim h'$ or

(b) $g \sim g'$ and $h = h'$.

For example, $K_2\Box K_2$ is isomorphic to C_4.

5. Show that $K_2\Box K_2\Box \ldots \Box K_2$, the product of d copies of K_2, is isomorphic to the hypercube graph Q_d, defined in HW #11.

6. Show that for every pair of graphs G, H, we have
$$\chi(G\Box H) = \max\{\chi(G), \chi(H)\}.$$